

On the Effect of Asymmetric Strange Seas and Isospin-Violating Parton Distribution Functions on $\sin^2 \theta_W$ Measured in the NuTeV Experiment

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The NuTeV collaboration recently reported a value of $\sin^2 \theta_W$ measured in neutrino-nucleon scattering that is 3 standard deviations above the standard model prediction. This result is derived assuming that (1) the strange sea is quark-antiquark symmetric, $s(x) = \bar{s}(x)$, and (2) up and down quark distributions are symmetric under the simultaneous interchange of $u \leftrightarrow d$ and $p \leftrightarrow n$. We report the impact of violations of these symmetries on $\sin^2 \theta_W$ and discuss the theoretical and experimental constraints on such asymmetries.

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I. INTRODUCTION AND FORMALISM

Based on measurements of neutral current and charged current neutrino-nucleon scattering in both neutrino and anti-neutrino beams, the NuTeV collaboration recently reported a measurement of $\sin^2 \theta_W^{(\text{on-shell})}$. The result [1],

$$\begin{aligned} \sin^2 \theta_W^{(\text{on-shell})} &= 0.2277 \pm 0.0013(\text{stat.}) \pm 0.0009(\text{syst.}) \\ &- 0.00022 \times \left(\frac{M_{\text{top}}^2 - (175 \text{ GeV})^2}{(50 \text{ GeV})^2} \right) \\ &+ 0.00032 \times \ln\left(\frac{M_{\text{Higgs}}}{150 \text{ GeV}}\right), \end{aligned} \quad (1)$$

is approximately 3 standard deviations above the expected value of 0.2227 ± 0.0004 [2, 3].

Ratios of neutral current to charged current cross sections on isoscalar targets of u and d quarks are experimental observables that can be related to fundamental electroweak parameters. Before NuTeV, high statistics neutrino experiments measured $\sin^2 \theta_W$ using the Llewellyn Smith cross section ratios [4]:

$$R^{\nu(\bar{\nu})} \equiv \frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma(\bar{\nu} N \rightarrow \ell^{-(+)} X)} = g_L^2 + r^{(-1)} g_R^2, \quad (2)$$

where

$$r \equiv \frac{\sigma(\bar{\nu} N \rightarrow \ell^+ X)}{\sigma(\bar{\nu} N \rightarrow \ell^- X)} \sim \frac{1}{2}, \quad (3)$$

and

$$\begin{aligned} g_L^2 &= (\epsilon_L^u)^2 + (\epsilon_L^d)^2 \\ &= \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W, \\ g_R^2 &= (\epsilon_R^u)^2 + (\epsilon_R^d)^2 \\ &= \frac{5}{9} \sin^4 \theta_W. \end{aligned} \quad (4)$$

For the experimental values of r and $\sin^2 \theta_W$, it follows that R^ν is much more sensitive to $\sin^2 \theta_W$ than is $R^{\bar{\nu}}$.

Inspired by the Paschos-Wolfenstein relationship [5]:

$$\begin{aligned} R^- &\equiv \frac{\sigma(\nu_\mu N \rightarrow \nu_\mu X) - \sigma(\bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \rightarrow \mu^- X) - \sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)} \\ &= \frac{R^\nu - r R^{\bar{\nu}}}{1 - r} = g_L^2 - g_R^2, \end{aligned} \quad (5)$$

NuTeV uses high statistics separated neutrino and anti-neutrino beams to measure $\sin^2 \theta_W$ and thereby reduces its sensitivity to uncertainties in cross sections resulting from scattering off $q\bar{q}$ symmetric quark seas. Using the separate neutrino and antineutrino data sets, NuTeV also extracts effective neutral current quark couplings, $(g_L^{\text{eff}})^2$ and $(g_R^{\text{eff}})^2$ [1].

Let $\langle q(x) \rangle$ denote the momentum distribution of a particular flavor of quark averaged over the nucleons in the NuTeV target, and let $\langle Q \rangle \equiv \int \langle q(x) \rangle dx$, the total momentum carried by quark flavor q . Let nucleon-specific quark momentum distributions be denoted by $q_p(x)$ and $q_n(x)$, with corresponding integrals Q_p and Q_n , respectively. Both the Llewellyn Smith and Paschos-Wolfenstein relationships assume $\langle U \rangle = \langle D \rangle$

and $\langle \bar{U} \rangle = \langle \bar{D} \rangle$. The Llewellyn Smith interpretation of R^ν assumes additionally that $\langle S \rangle = \langle \bar{S} \rangle = \langle C \rangle = \langle \bar{C} \rangle$ (clearly $\langle S \rangle = \langle C \rangle$ is experimentally not a good assumption), while the Paschos-Wolfenstein R^- formula assumes only $\langle S \rangle = \langle \bar{S} \rangle$ and $\langle C \rangle = \langle \bar{C} \rangle$.

The NuTeV $\sin^2 \theta_W$ analysis accounts for the violations of the assumption that $\langle u(x) \rangle = \langle d(x) \rangle$ and $\langle \bar{u}(x) \rangle = \langle \bar{d}(x) \rangle$ which result from the excess of neutrons over protons in the target. From a material inventory of the NuTeV target calorimeter, we measure a $5.74 \pm 0.02\%$ fractional excess of neutrons over protons [6]. However, the NuTeV result assumes exact isospin symmetry in neutron and proton quark distributions, $u_p^{(-)}(x) = d_n^{(-)}(x)$, $d_p^{(-)}(x) = u_n^{(-)}(x)$. The NuTeV analysis assumes furthermore that $\langle s(x) \rangle = \langle \bar{s}(x) \rangle$ and $\langle c(x) \rangle = \langle \bar{c}(x) \rangle$. It has been pointed out that such assumptions, if incorrect, produce sizable shifts in the NuTeV $\sin^2 \theta_W$ [7, 8, 9, 10].

Although the NuTeV experiment does not exactly measure R^- , in part because it is not possible experimentally to measure neutral current reactions down to zero recoil energy, it is nevertheless illustrative to calculate the effect of these violations on R^- . Denote the neutron excess of the NuTeV target as $\delta N \equiv (A - 2Z)/A$ and the total valence momentum carried by the proton as $V_p = U_p - \bar{U}_p + D_p - \bar{D}_p$. Let the following

$$\begin{aligned} \delta D_v &\equiv D_p - \bar{D}_p - U_n + \bar{U}_n \\ \delta U_v &\equiv U_p - \bar{U}_p - D_n + \bar{D}_n \\ \delta \bar{D} &\equiv \bar{D}_p - \bar{U}_n \\ \delta \bar{U} &\equiv \bar{U}_p - \bar{D}_n \\ \delta S &\equiv \langle S \rangle - \langle \bar{S} \rangle \end{aligned} \quad (6)$$

denote deviations from the above symmetry assumptions. To first order in δN , δQ_v , $\delta \bar{Q}$ and δS , we obtain

$$\begin{aligned} R^- &\approx \Delta_u^2 + \Delta_d^2 \\ &\quad - \delta N \left(\frac{U_v - D_v}{V_p} \right) (3\Delta_u^2 + \Delta_d^2) \\ &\quad + \frac{\delta U_v - \delta D_v}{2V_p} (3\Delta_u^2 + \Delta_d^2) \\ &\quad + \frac{\delta S}{V_p} (2\Delta_d^2 - 3(\Delta_d^2 + \Delta_u^2)\epsilon_c), \end{aligned} \quad (7)$$

where $\Delta_{u,d}^2 = (\epsilon_L^{u,d})^2 - (\epsilon_R^{u,d})^2$ and where ϵ_c denotes the ratio of the scattering cross section from the strange sea including kinematic suppression of heavy charm production to that without kinematic suppression. In this calculation, we assume the massless quark-parton model which implies no longitudinal cross section, no target mass effects, and we also assume $\langle C \rangle = \langle \bar{C} \rangle = 0$.

As already noted, to extract $\sin^2 \theta_W$, NuTeV does not measure directly R^- , but rather measures ratios of experimental candidates within kinematic criteria and compares this to a full Monte Carlo simulation which accounts for neutral current and charged current cross-talk, non-quark-parton model

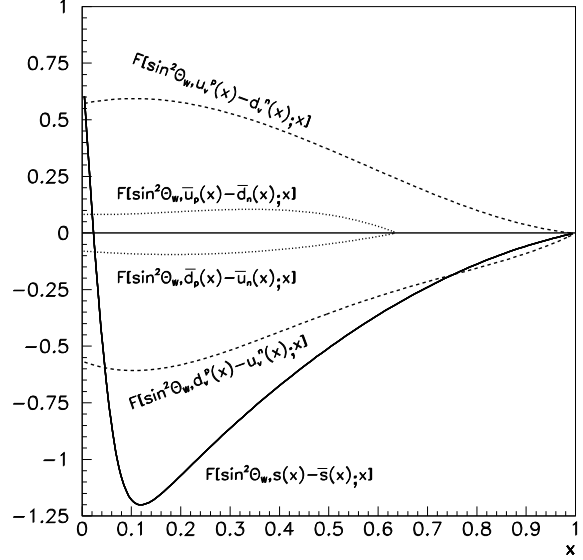


FIG. 1: The functionals describing the shift in the NuTeV $\sin^2 \theta_W$ caused by not correcting the NuTeV analysis for isospin violating u and d valence and sea distributions or for $\langle s(x) \rangle \neq \langle \bar{s}(x) \rangle$. The shift in $\sin^2 \theta_W$ is determined by convolving the asymmetric momentum distribution with the plotted functional.

contributions to the cross section, radiative corrections, electron neutrino backgrounds, and detector resolution [1]. Therefore, the NuTeV $\sin^2 \theta_W$ measurement does not depend on these symmetry violating terms in the way that Equations 5 and 7 would suggest.

To examine the exact effect of various symmetry violations on the NuTeV analysis, we first define a functional $F[\mathcal{E}, \delta; x]$ such that the shift in an experimental quantity, \mathcal{E} , due to a symmetry violating quark fractional momentum distribution, $\delta(x)$, is given by:

$$\Delta \mathcal{E} = \int_0^1 F[\mathcal{E}, \delta; x] \delta(x) dx. \quad (8)$$

All of the details of the NuTeV Monte Carlo simulation and measurement can be parameterized in terms of $F[\mathcal{E}, \delta; x]$, and therefore, this formalism provides a way to determine the shift in the NuTeV measurement for arbitrary symmetry violation in PDFs. Figures 1 and 2 show $F[\mathcal{E}, \delta; x]$ for an isospin symmetry violating u and d valence and sea and for $\langle s(x) \rangle \neq \langle \bar{s}(x) \rangle$. Figure 1 shows the functionals for the NuTeV measurement of $\sin^2 \theta_W$, while Figure 2 shows the corresponding functionals for $(g_L^{\text{eff}})^2$ and $(g_R^{\text{eff}})^2$.

II. ASYMMETRIC STRANGE SEA

If the strange sea is generated by purely perturbative QCD processes, then neglecting electromagnetic effects, one expects $\langle s(x) \rangle = \langle \bar{s}(x) \rangle$. However, it has been noted that

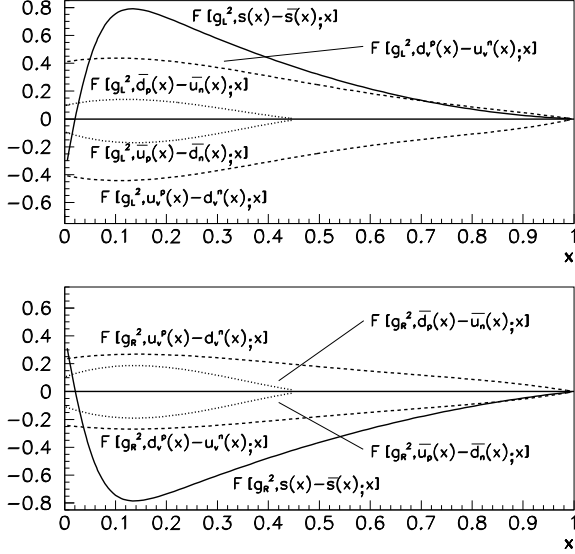


FIG. 2: The functionals describing the shifts in the NuTeV $(g_L^{\text{eff}})^2$ and $(g_R^{\text{eff}})^2$ caused by not correcting the NuTeV analysis for isospin violating u and d valence and sea distributions or for $\langle s(x) \rangle \neq \langle \bar{s}(x) \rangle$. The shifts in $(g_L^{\text{eff}})^2$ and $(g_R^{\text{eff}})^2$ are determined by convolving the asymmetric momentum distribution with the plotted functional.

non-perturbative QCD effects can generate a significant momentum asymmetry between the strange and anti-strange seas [11, 12, 13, 14]. Lending weight to this possibility, a joint fit to CDHS neutrino charged-current inclusive cross sections [16] (but not including CCFR [17] and NuTeV data or neutrino dimuon cross sections) and charged lepton structure function data reports some improvement in their fits if they allow for an asymmetry in the strange sea at high x [15]. The CCFR and CDHS charged current neutrino cross-sections differ significantly at high x where this joint fit finds a large strange sea asymmetry, $s \gg \bar{s}$.

By measuring the processes $\nu_N, \bar{\nu}_N \rightarrow \mu^+ \mu^- X$ the CCFR and NuTeV experiments constrain the difference between the momentum distributions of the strange and anti-strange seas. For studying the effect on the NuTeV $\sin^2 \theta_W$, it is important to study such effects within the same PDF formalism and corresponding cross sections as were used in the measurement itself [1]. In this enhanced leading order cross section model, the CCFR/NuTeV $\nu, \bar{\nu}$ dimuon data were fit [18] to the following form for the strange and anti-strange seas [23]:

$$\begin{aligned} \langle s(x) \rangle &= \kappa \frac{\langle \bar{u}(x) \rangle + \langle \bar{d}(x) \rangle}{2} (1-x)^\alpha \\ \langle \bar{s}(x) \rangle &= \bar{\kappa} \frac{\langle \bar{u}(x) \rangle + \langle \bar{d}(x) \rangle}{2} (1-x)^{\bar{\alpha}}, \end{aligned} \quad (9)$$

obtaining central values of

$$\begin{pmatrix} \kappa \\ \bar{\kappa} \\ \alpha \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} .352 \\ .405 \\ -0.77 \\ -2.04 \end{pmatrix} \quad (10)$$

and a covariance matrix[24] incorporating both statistical and systematic uncertainties on these parameters:

$$\begin{pmatrix} 0.0034 & 0.0027 & -0.028 & -0.007 \\ 0.0027 & 0.0031 & -0.024 & -0.008 \\ -0.028 & -0.024 & 0.78 & 0.18 \\ -0.007 & -0.008 & 0.18 & 0.29 \end{pmatrix}. \quad (11)$$

Within this particular model, the measurement implies a *negative* asymmetry,

$$\langle S \rangle - \langle \bar{S} \rangle = -0.0027 \pm 0.0013, \quad (12)$$

and a resulting increase in the NuTeV value of $\sin^2 \theta_W$,

$$\Delta \sin^2 \theta_W = +0.0020 \pm 0.0009. \quad (13)$$

The initial NuTeV measurement, which assumes $\langle s(x) \rangle = \langle \bar{s}(x) \rangle$, becomes $\sin^2 \theta_W = 0.2297 \pm 0.0019$. Hence, if we use the experimental measurement of the strange sea asymmetry, the discrepancy with the standard model is increased to 3.7σ significance.

A recent calculation [10] claims that a *positive* strange sea asymmetry of $\langle S \rangle - \langle \bar{S} \rangle = +0.0020$ could explain half of the NuTeV discrepancy ($\Delta \sin^2 \theta_W = -0.0026$). It should be noted, however, that this is an overestimate, as Figure 1 makes clear, due to the fact that charged current charm suppression threshold effects have been neglected in their analysis, and because NuTeV does not exactly measure R^- [25].

Reference [15] reports favoring a significant positive strange sea asymmetry ($S - \bar{S} \sim +0.0020$) at high x . A fit to the form assumed in Equation 9 does not necessarily exclude such an asymmetry as it is dominated by data at low x . The asymmetry of Reference [15] would imply at least a 5% increase in the total ν dimuon cross section in the region $x > 0.5$. However, NuTeV has looked for such an excess at high x and excludes additional dimuon sources larger than 0.2% (0.6%) in the ν ($\bar{\nu}$) data at 90% confidence [18].

III. ISOSPIN VIOLATING PDFS

Several recent classes of non-perturbative models predict isospin violation in the nucleon [7, 8, 9]. We evaluate the shift in the NuTeV value of $\sin^2 \theta_W$ under the assumption that the asymmetry occurs in nature and is not corrected for in the NuTeV analysis. The earliest estimation in the literature, a bag model calculation [7], predicts large valence asymmetries of opposite sign in $u_p - d_n$ and $d_p - u_n$ at all x , which would produce a shift in the NuTeV $\sin^2 \theta_W$ of -0.0020 . However, this estimate neglects a number of effects, and a complete calculation by Thomas *et al.* [8] concludes that asymmetries at very high x are larger, but the asymmetries at moderate x are

smaller and of opposite sign at low x , thereby reducing the shift in $\sin^2 \theta_W$ to a negligible -0.0001 . Finally, the effect is also evaluated in the Meson Cloud model [9], and there the asymmetries are much smaller at all x , resulting in a modest shift in the NuTeV $\sin^2 \theta_W$ of $+0.0002$.

The calculation of Thomas *et al.* [8] is particularly useful in evaluating uncertainties because it decomposes isospin violating effects into different parts that are driven by experimental or theoretical inputs. The largest contributions to a shift in $\sin^2 \theta_W$ in this calculation come from the single quark ($m_d - m_u \sim 4$ MeV) and nucleon ($m_n - m_p \approx 1.29$ MeV) mass differences. The former has a significant theoretical uncertainty, and we assign a fractional error of 25% to this source of isospin violation based on the uncertainty in $m_d - m_u$ [19, 20]; such an uncertainty translates to a 0.0001 uncertainty in the NuTeV $\sin^2 \theta_W$. Another contribution in this calculation with large theoretical uncertainties is the effect of diquark ($m_{dd} - m_{uu}$) mass differences. This causes isospin breaking predominantly at high x where both the PDFs are small and the effect on the NuTeV measurement is negligible. The uncertainty is therefore significantly smaller than that from the single quark mass shift.

In general, nuclear effects can also cause isospin-breaking, thereby producing $\langle U \rangle \neq \langle D \rangle$ in the NuTeV target, which is primarily iron. While less theoretically certain, one estimate of the effect exists [21] and would predict a modest increase in the NuTeV $\sin^2 \theta_W$.

Although a particular nucleon or nuclear charge symmetry violation model could account for the NuTeV discrepancy with the standard model, such models, in their attempt to ex-

plain the NuTeV $\sin^2 \theta_W$, must be evaluated in the context of a global fit to all experimental data derived from any such asymmetry assumptions because they may disagree with existing data [22].

IV. CONCLUSIONS

The fact that NuTeV does not measure directly R^- or exact ratios of neutral to charged current cross sections makes it difficult to predict the effect of parton level symmetry violations. Hence, we present a framework for evaluating the effects of both isospin violating u and d parton densities and asymmetric strange seas on the NuTeV measurements of $\sin^2 \theta_W$, $(g_L^{\text{eff}})^2$, and $(g_R^{\text{eff}})^2$. While it is possible, in principle, to induce sizable shifts in the NuTeV $\sin^2 \theta_W$ with variations in the former, the joint CCFR/NuTeV neutrino and anti-neutrino dimuon data limit possible charge asymmetry in the strange sea. In fact, relaxing the restriction that $\langle s(x) \rangle = \langle \bar{s}(x) \rangle$ in the LO fit to CCFR/NuTeV dimuon data increases the NuTeV discrepancy with the standard model.

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 - [23] At $Q^2 = 16 \text{ GeV}^2$, the average Q^2 of the NuTeV data used in the $\sin^2 \theta_W$ analysis, the NuTeV $(\langle \bar{u}(x) \rangle + \langle \bar{d}(x) \rangle)/2$ can be parameterized as $e^{-0.75-150x} + e^{-1.33-7.7x-8.1x^2}$ over the region $0 < x < 0.6$. NuTeV determines its leading order PDFs from fits to CCFR cross section data including external constraints [1, 17].
 - [24] This covariance matrix is from the fit of Ref. 18, although the matrix is not given in the original paper.
 - [25] The inclusion of an asymmetric strange sea induces a larger and opposite sign shift in $R^{\overline{\nu}}$ compared to the shift in R^{ν} . Because the NuTeV result is less sensitive to $R^{\overline{\nu}}$ than is R^- , the effect is reduced at all x . The large suppression of charged-current scattering from the low x strange sea explains the change of sign in the shift in $\sin^2 \theta_W$ at very low x .